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- Thus, DFAs and Regular Expressions are equivalent.

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- $R^+ = RR^*$ and R^k for k-fold concatenation are useful shorthands.

Regular Expression Examples

Regular Expression		Regular Language
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$(0^*10^*1)^*0^*$	$\rightarrow$	$\{\omega \mid n_1(\omega)$ is even$\}$

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- Note that we do not have explicit operators for intersection or complementation!

Converting REs to NFAs: Basis Cases

Regular Expression **Corresponding NFA**

ϕ

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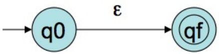
ϕ

ϵ



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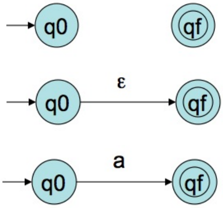
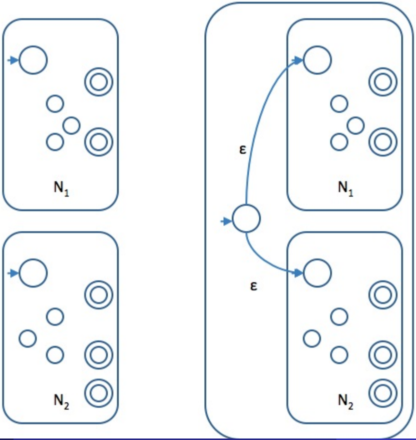
a for $a \in \Sigma$



Converting REs to NFAs

Union

- Let N_1 and N_2 be NFAs for R_1 and R_2 respectively. Then the NFA for $\mathbf{R_1 \cup R_2}$ is



Concatenation

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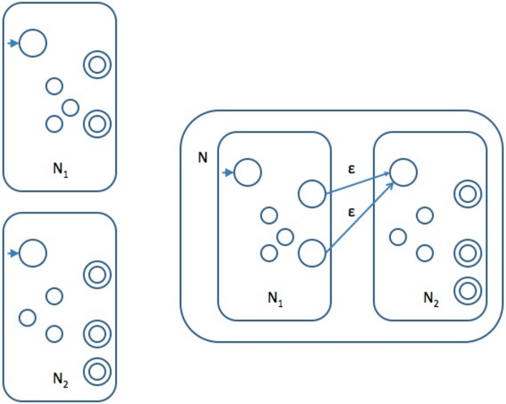
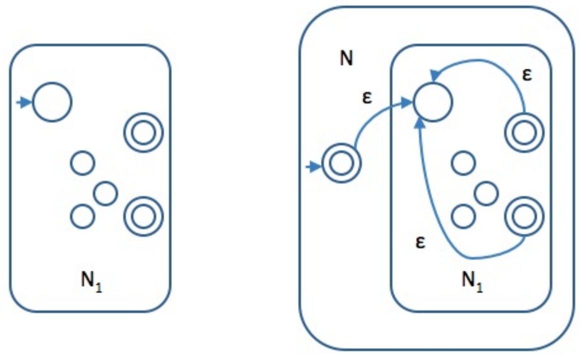


Star

- Let N be NFAs for R. Then the NFA for $\mathbf{R}^*$ is

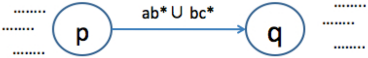


RE to NFA Conversion Example

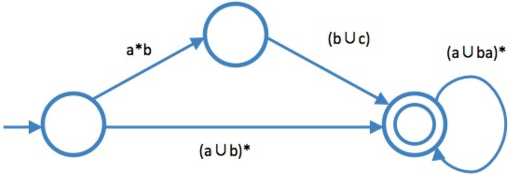
- Let's convert $(\mathbf{a} \cup \mathbf{b})^*\mathbf{aba}$ to an NFA.

Generalized Transitions and NFA

- A generalized transition is a transition whose label is a regular expression



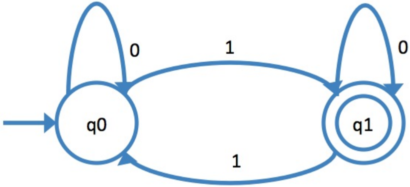
- A Generalized NFA is an NFA with generalized transitions.



  - In fact, all standard DFA transitions are generalized transitions with regular expressions of a single symbol!

GENERALIZED TRANSITIONS

- Consider the 2-state DFA



$\downarrow$

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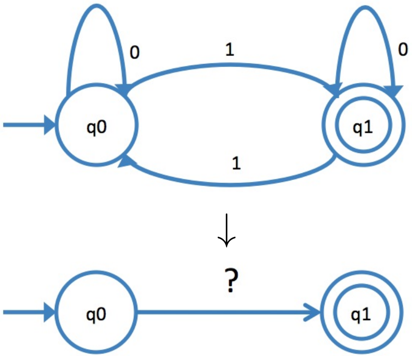

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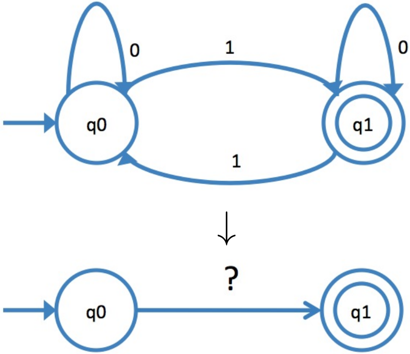

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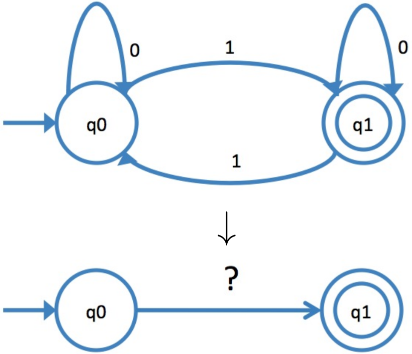

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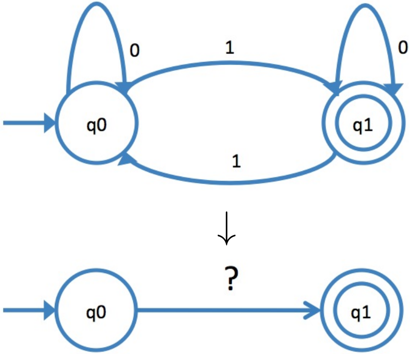

- $\mathbf{0}^*\mathbf{1}$ takes the DFA from state q_0 to q_1
- $(\mathbf{0} \cup \mathbf{10}^*\mathbf{1})^*$ takes the machine from q_1 back to q_1
- So $\mathbf{?} = \mathbf{0}^*\mathbf{1}(\mathbf{0} \cup \mathbf{10}^*\mathbf{1})^*$ represents all strings that take the DFA from state q_0 to q_1